

On drag forces and jet quenching in strongly-coupled plasmas

Elena Cáceres^a and Alberto Güijosa^b

^a*Facultad de Ciencias, Universidad de Colima
Bernal Díaz del Castillo 340, Colima, Colima, México*

^b*Departamento de Física de Altas Energías
Instituto de Ciencias Nucleares, Universidad Autónoma de México
Apdo. Postal 70-543, México D.F.04510, México
E-mail: elenac@uacol.mx, alberto@nucleares.unam.mx*

ABSTRACT: We compute the drag force experienced by a heavy quark that moves through plasma in a gauge theory whose dual description involves arbitrary metric and dilaton fields. As a concrete application, we consider the cascading gauge theory at temperatures high above the deconfining scale, where we obtain a drag force with a non-trivial velocity dependence. We compare our results with the jet-quenching parameter for the same theory, and find qualitative agreement between the two approaches. Conversely, we calculate the jet-quenching parameter for $\mathcal{N} = 4$ super-Yang-Mills with an R-charge density (or equivalently, a chemical potential), and compare our result with the corresponding drag force.

KEYWORDS: AdS-CFT Correspondence, Gauge-gravity correspondence.

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1. Introduction and summary

Recently there has been a surge of interest in the possibility of employing the gauge/gravity duality [1, 2] to determine the rate of energy loss in finite-temperature strongly-coupled gauge theories. The ultimate aim of this program would be to make contact with current [3] and future [4] experimental studies of quark-gluon plasma (QGP), but at this point the gravity dual of QCD is not yet available, so one must still be cautious when attempting to draw inferences in this direction.

In the AdS/CFT context, the issue of energy loss has been approached from three different perspectives. The authors of [5] proposed a model-independent, non-perturbative definition of the jet-quenching parameter (which in the QGP case codifies the suppression of hadrons with high transverse momenta) in terms of a light-like Wilson loop, which they then computed in $\mathcal{N} = 4$ super-Yang-Mills (SYM) using the recipe of [6]. Their computation was generalized in [7] to the cascading gauge theory [8, 9] at finite temperature [10], and in [11] to certain marginal deformations of the $\mathcal{N} = 4$ theory. Related work may be found in [12, 13].

A second approach was pursued in [14, 15], where the drag force experienced by a heavy quark that moves through $\mathcal{N} = 4$ SYM plasma was determined by considering a string in the dual AdS-Schwarzschild geometry. This calculation was extended to all asymptotically AdS geometries in [16], including the case dual to $\mathcal{N} = 4$ SYM with a non-zero chemical potential, a case that was studied simultaneously in [17] from a different perspective: whereas [17] worked directly with the ten-dimensional spinning D3-brane background, [16] employed instead the five-dimensional charged black hole solution of $\mathcal{N} = 8$ gauged supergravity [18] obtained upon Kaluza-Klein reduction on the S^5 [19], thereby arriving at different results. A more detailed picture of the energy-loss process was painted in [20], which studied the wake left by the quark as it ploughs through the plasma, using the methods of [21, 22]. The connection between the rate of energy loss found in [14, 15] and magnetic confinement was explored very recently in [23].

A third approach [24] extracted the diffusion coefficient for a heavy quark in the $\mathcal{N} = 4$ plasma from an analysis of small fluctuations of a Wilson line that follows the Schwinger-Keldysh contour. On the AdS side of the duality, this involved a study of fluctuations that propagate along a string; similar calculations were carried out in [14].

It is important to explore the relation between these three approaches. As a step in this direction, in this paper we compare the drag forces and jet-quenching parameters for two different gauge theories. We begin in section 2 by generalizing the drag force calculation of [14–16] to backgrounds with arbitrary metric and dilaton fields, finding the general result (2.12). In section 3 we then specialize to the cascading gauge theory, where the resulting drag force (3.4) is found to display a highly non-trivial velocity dependence. We compare this force with the jet-quenching parameter¹ (3.9) determined in [7], finding agreement in functional form, and numerical agreement (up to an overall constant) in the region of large velocities. In section 4 we proceed in the opposite direction, computing the jet-quenching parameter for the $\mathcal{N} = 4$ plasma with an R-charge density, and comparing the result with the drag force determined in [17]. This comparison is interesting because in this case the quantities to be compared are not just numbers but functions of the charge density J . For small values of J , our result (4.15) is again in qualitative agreement with the drag force (4.16) obtained in [17], but for arbitrary charges, the results (4.12) and (4.19) disagree.

The general lesson appears to be that the parameter \hat{q} defined by [5] and the dissipative force extracted from the procedure pioneered in [14, 15] represent closely related but not identical measures of the rate of energy dissipation in a given non-Abelian plasma. Our results underline the interest in exploring the connection between the various approaches to energy loss from a more general viewpoint, by attempting to extrapolate from one to the other directly at the level of the corresponding AdS/CFT calculations.²

2. Drag force in gauge theories with holographic duals

Consider a background dilaton field $\phi(x)$ and stationary Einstein frame metric

$$ds_E^2 = G_{\mu\nu}(x) dx^\mu dx^\nu \tag{2.1}$$

that holographically encode the dynamics of a strongly-coupled gauge theory. Since we are interested in studying this gauge theory at a finite temperature T , we assume the geometry (2.1) includes a black hole [28]. In this setup one can introduce an external quark in the gauge theory by considering a string that has a single endpoint at the boundary and extends all the way down to the horizon [6] (the gauge theory is therefore non-confining at the given temperature).

¹To be more precise, we find that the directly comparable quantities are the jet-quenching parameter \hat{q} and the ratio μ/T , where μ is the friction coefficient that determines the drag force through $dp/dt = -\mu p$.

²After the first version of this paper had appeared on the arXiv, the relation between the drag force [14, 15] and jet-quenching [5] approaches was discussed in the context of a study of mesons moving through the plasma [25, 26]. Related work may be found in [27].

The relevant string dynamics are captured by the Nambu-Goto action

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma e^{\phi/2} \sqrt{-\det g_{\alpha\beta}}, \quad (2.2)$$

with $g_{\alpha\beta} = G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu$. Letting z denote the radial coordinate of the black hole geometry and t, x^i ($i = 1, 2, 3$) label the directions along the boundary at spatial infinity, we make the static gauge choice $\sigma = z, \tau = t$, and, following [14, 15], focus attention on the configuration

$$X^1(z, t) = vt + \rho(z), \quad X^2 = 0 = X^3. \quad (2.3)$$

For the appropriate sign of ρ' , this describes the string trailing behind its boundary endpoint as it moves at constant speed v in the x^1 direction, a configuration dual to the external quark traversing the plasma.

Using (2.3) in (2.2), we find the Lagrangian

$$\mathcal{L} \equiv e^{\phi/2} \sqrt{-\det g} = e^{\phi/2} \sqrt{-G_{zz}G_{tt} - G_{zz}G_{xx}v^2 - G_{xx}G_{tt}\rho'^2}, \quad (2.4)$$

which results in an equation of motion for ρ implying that

$$\pi_X = \frac{\partial \mathcal{L}}{\partial \rho'} = e^{\phi/2} \frac{G_{xx}G_{tt}}{\sqrt{-g}} \rho' \quad (2.5)$$

is a constant. Inverting this relation we obtain

$$(\rho')^2 = -\pi_X^2 \frac{G_{zz}(G_{tt} + G_{xx}v^2)}{G_{xx}G_{tt}(e^\phi G_{xx}G_{tt} + \pi_X^2)}. \quad (2.6)$$

Just like in the $\mathcal{N} = 4$ case analyzed in [14, 15], for $v^2 > 0$ the numerator in this expression changes sign at a radius $z = z_v$ defined by

$$(G_{tt} + G_{xx}v^2)|_{z=z_v} = 0, \quad (2.7)$$

and so the string will not be able to extend all the way down to the horizon at $z = z_H$ unless the denominator also changes sign at z_v . This condition fixes

$$\pi_X^2 = -e^\phi G_{xx}G_{tt}|_{z=z_v} = v^2 e^\phi G_{xx}^2|_{z=z_v}. \quad (2.8)$$

With (2.6) we can then compute the current density for momentum along x^1 ,

$$P_x^z = -\frac{1}{2\pi l_s^2} e^{\phi/2} G_{x\nu} g^{z\alpha} \partial_\alpha X^\nu \quad (2.9)$$

$$= -\frac{1}{2\pi l_s^2} e^{\phi/2} \frac{G_{xx}G_{tt}}{\det g} \rho', \quad (2.10)$$

and use it to compute the drag force experienced by the string/quark,

$$\frac{dp_1}{dt} = \sqrt{-\det g} P_x^z = -\frac{1}{2\pi l_s^2} e^{\phi/2} \frac{G_{xx}G_{tt}}{\sqrt{-\det g}} \rho', \quad (2.11)$$

which after some algebra is easily seen to simplify to

$$\frac{dp_1}{dt} = -\frac{\pi_X}{2\pi l_s^2} = -\frac{v}{2\pi l_s^2} e^{\phi/2} G_{xx}|_{z=z_v}. \quad (2.12)$$

This generalizes the result obtained in [16] to backgrounds with an arbitrary dilaton profile.

3. Drag force in the cascading plasma

Let us now apply the results of the previous section to an interesting concrete example: the Klebanov-Strassler cascading gauge theory [8, 9], whose dual geometry at temperatures high above the deconfining transition was constructed in [10] (see also [33, 32]) and is given by

$$\begin{aligned}
 ds^2 &= \frac{\sqrt{8a/K_*}}{\sqrt{z}} e^{2P^2\eta} \left(-(1-z)dt^2 + d\vec{X}^2 \right) + \frac{\sqrt{K_*}}{32} e^{-2P^2(\eta-5\xi)} \frac{dz^2}{z^2(1-z)} \\
 &\quad + \frac{\sqrt{K_*}}{2} e^{-2P^2(\eta-\xi)} \left[e^{-8P^2\omega} e_\psi^2 + e^{2P^2\omega} (e_{\theta_1}^2 + e_{\phi_1}^2 + e_{\theta_2}^2 + e_{\phi_2}^2) \right], \\
 \xi &= \frac{2z + [-2z + (z-2)\log(1-z)]\log z + (z-2)\text{Li}_2(z)}{40K_*z}, \\
 \eta &= \frac{z-2}{16K_*z} [\log z \log(1-z) + \text{Li}_2(z)], \\
 \phi &= \frac{P^2}{K_*} \left(-\frac{\pi^2}{24} + \frac{1}{4}\text{Li}_2(1-z) \right),
 \end{aligned} \tag{3.1}$$

where the radial coordinate z runs from the horizon at $z = 1$ to the boundary at $z \rightarrow 0$.

Using (3.1) in (2.8) one finds that $z_v = 1 - v^2$ and

$$\pi_X = -v \left(e^{\phi/2} G_{xx} \right)_{z=z_v} = -\sqrt{\frac{8a}{K_*}} v \left(\frac{e^{2P^2\eta} e^{\phi/2}}{\sqrt{z}} \right)_{z=1-v^2}, \tag{3.2}$$

or, more explicitly,

$$\begin{aligned}
 \pi_X &= -\sqrt{\frac{8a}{K_*}} \frac{v}{\sqrt{1-v^2}} \exp \left\{ \frac{P^2}{K_*} f(v) \right\}, \\
 f(v) &\equiv \frac{1}{2} \left[-\frac{\pi^2}{24} + \frac{1}{4}\text{Li}_2 v^2 - \frac{(1+v^2)}{4(1-v^2)} (\log(1-v^2)\log v^2 + \text{Li}_2(1-v^2)) \right].
 \end{aligned} \tag{3.3}$$

The drag force is then given by (2.12) as

$$\frac{dp}{dt}_{KT} = -\frac{1}{2\pi l_s^2} \sqrt{\frac{8a}{K_*}} \frac{v}{\sqrt{1-v^2}} \left[1 + f(v) \frac{P^2}{K_*} + \mathcal{O} \left(\frac{P^4}{K_*^2} \right) \right], \tag{3.4}$$

where we have kept only the first two terms in an expansion in powers of $P^2/K_* \ll 1$, because the solution (3.1) itself is only valid to this order. The velocity-dependence seen in the first term is just the p/m factor present already in $\mathcal{N} = 4$ SYM [14, 15]. The second term has an additional non-trivial dependence codified in the function $f(v)$ defined in (3.3), which approaches the value $-\pi^2/24 \simeq -0.411$ for $v \rightarrow 0$, and has a logarithmic divergence in the ultrarelativistic region $v \rightarrow 1$. As seen in figure 1, away from this region $f(v)$ is nearly constant.

As in [7], it is instructive to compare the result (3.4) for the cascading plasma against the drag force in $\mathcal{N} = 4$ SYM [14, 15],

$$\frac{dp}{dt}_{\mathcal{N}=4} = -\frac{\pi \sqrt{g_{YM}^2 N}}{2} T^2 \frac{v}{\sqrt{1-v^2}}, \tag{3.5}$$

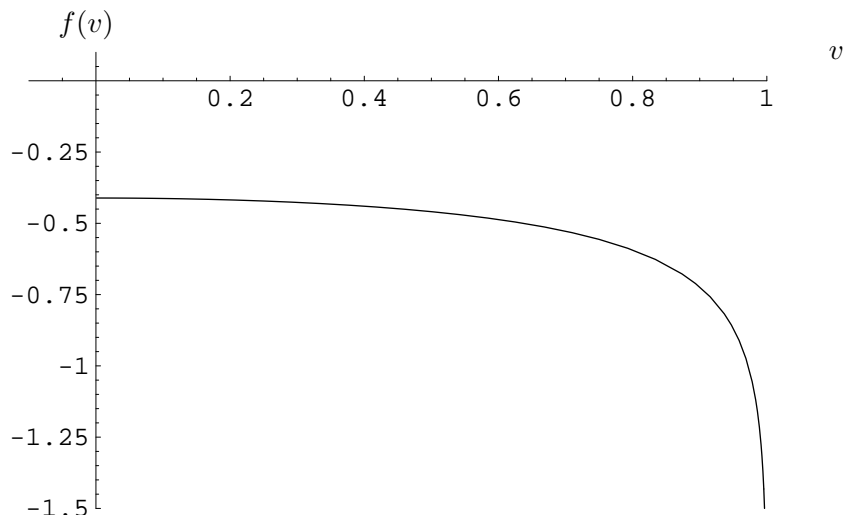


Figure 1: The function defined in (3.3), which codifies the dependence of the drag force (3.4) on the velocity beyond the expected relativistic dependence $v/\sqrt{1-v^2}$ found already in $\mathcal{N} = 4$ SYM.

by considering the ratio

$$\frac{\frac{dp}{dt}KT}{\frac{dp}{dt}\mathcal{N}=4} = \frac{1}{\pi^2 l_s^2} \sqrt{\frac{8a}{K_*}} \frac{1}{T^2} \frac{1}{\sqrt{g_{YM}^2 N}} \left[1 + \frac{P^2}{K_*} f(v) + \mathcal{O}\left(\frac{P^4}{K_*^2}\right) \right]. \quad (3.6)$$

Employing the relation $8a = T^4 K_*^2/4$ [10], this reads

$$\frac{\frac{dp}{dt}KT}{\frac{dp}{dt}\mathcal{N}=4} = \frac{1}{2\pi^2 l_s^2} \sqrt{\frac{K_*}{g_{YM}^2 N}} \left[1 + \frac{P^2}{K_*} f(v) + \mathcal{O}\left(\frac{P^4}{K_*^2}\right) \right]. \quad (3.7)$$

Upon synchronizing the rank of the $\mathcal{N} = 4$ theory with the effective rank of the cascading gauge theory at the given temperature by setting $K_* = \sqrt{2/\pi} L_P^4 N = \sqrt{4\kappa^2 \pi} N = 2^4 \pi^3 g_s l_s^4 N$, and using $g_{YM}^2 = 4\pi g_s$, we are left with the final result

$$\frac{\frac{dp}{dt}KT}{\frac{dp}{dt}\mathcal{N}=4} = 1 + f(v) \frac{P^2}{K_*} + \mathcal{O}\left(\frac{P^4}{K_*^2}\right). \quad (3.8)$$

This ratio has the same qualitative form as the one computed for the corresponding jet-quenching parameter calculated in [7],

$$\frac{\hat{q}_{cascade}}{\hat{q}_{\mathcal{N}=4}} = 1 + \chi \frac{P^2}{K_*} + \mathcal{O}\left(\frac{P^4}{K_*^2}\right), \quad (3.9)$$

with $\chi \simeq -1.388$. In both cases the temperature-dependence arises only from the dependence of the effective rank K_* on T , the precise form of which is $K_*/2P^2 \simeq \log(T/\Lambda)$ [10]. In addition, $f(v)$ is negative for any value of v (see figure 1), which implies that, just like the jet-quenching parameter, the drag force increases with increasing temperature. It also seems worth pointing out that precise numerical agreement between $f(v)$ and χ is achieved at a rather large value of the velocity, $v \sim .994$, which appears to be related to the fact that the jet-quenching calculation focuses on ultra-relativistic quarks.

Before closing this section we should note that it has recently been argued [34] that (3.1) cannot be trusted all the way down to the boundary at $z = 0$, because the perturbative expansion in $P^2/K_* \ll 1$ through which it was derived [10] breaks down at a critical radius $z_c = e^{-2K_*/P^2}$, which is non-perturbatively small but non-zero. Since the drag force (3.4) is determined by the value of the metric and dilaton at $z_v = 1 - v^2$, this does not affect our result as long as $z_v \gg z_c$, which means that we can consider ultra-relativistic velocities except for a region parametrically close to $v = 1$, defined by the condition $\ln \gamma \geq K_*/P^2$. As an example, validity of (3.4) at the velocity $v = 0.994$ considered in the preceding paragraph merely requires that $P^2/K_* \ll 0.45$. The problem of obtaining a solution valid all the way down to $z = 0$ has been addressed numerically in [34]. In any case, our main goal in this section has been to obtain a drag force comparable to the jet-quenching parameter (3.9), which was derived in [7] using the background (3.1).

4. Jet-quenching parameter in a charged $\mathcal{N} = 4$ plasma

The near-horizon metric for rotating D3-branes at finite temperature and with one angular momentum turned on is [30, 29, 31]

$$ds^2 = \frac{1}{\sqrt{H}} \left[\frac{(1-h)}{2} ((dx^+)^2 + (dx^-)^2) - (1+h) dx^+ dx^- + dx_2^2 + dx_3^2 \right] + \sqrt{H} \left[\frac{dr^2}{\tilde{h}} - \frac{2lr_0^2}{R^2} \sin^2 \theta dt d\phi + r^2 (\Delta d\theta^2 + \tilde{\Delta} \sin^2 \theta d\phi^2 + \cos^2 \theta d\Omega_3^2) \right], \quad (4.1)$$

where

$$\begin{aligned} x^\pm &= \frac{1}{\sqrt{2}}(x^0 \pm x^1), \\ H &= \frac{R^4}{r^4 \Delta}, \\ \Delta &= 1 + \frac{l^2 \cos^2 \theta}{r^2}, \\ \tilde{\Delta} &= 1 + \frac{l^2}{r^2}, \\ h &= 1 - \frac{r_0^4}{r^4 \Delta}, \\ \tilde{h} &= \frac{1}{\Delta} \left(1 + \frac{l^2}{r^2} - \frac{r_0^4}{r^4} \right), \\ R^4 &= 4\pi N g_s l_s^4. \end{aligned} \quad (4.2)$$

This background is dual to $\mathcal{N} = 4$ SYM theory with an R-charge density, or equivalently, a chemical potential. The geometry has an event horizon at the positive root of $\tilde{h}(r_H) = 0$,

$$r_H^2 = \frac{1}{2} \left(\sqrt{l^4 + 4r_0^4} - l^2 \right), \quad (4.3)$$

and an associated Hawking temperature, angular momentum density, and angular velocity at the horizon

$$T = \frac{r_H}{2\pi R^2 r_0^2} \sqrt{l^4 + 4r_0^4}, \quad J = \frac{lr_0^2 R^2}{64\pi^4 g_s^2 l_s^8}, \quad \Omega = \frac{lr_H^2}{r_0^2 R^2}, \quad (4.4)$$

which translate respectively into the temperature, R-charge density and R-charge chemical potential in the gauge theory.

In this section we will calculate the jet-quenching parameter \hat{q}_J following [5], and compare with the drag force result of [17]. For this we must consider a string whose endpoints lie at $r \rightarrow \infty$ in the spacetime (4.1) and trace a rectangular light-like Wilson loop of length L along $x^2 \equiv y$ and L^- along x^- [5]. Making the static gauge choice $\sigma = y, \tau = x^-$, the relevant configuration is $r(y, x^-) = r(y)$, with all other embedding fields constant, and with boundary conditions $r(\pm L/2) = \infty$. The Nambu-Goto action reduces to

$$S = \frac{\sqrt{2}L^-}{2\pi\alpha'} \int_0^{L/2} dy \sqrt{\left(\frac{1}{H} + \frac{(r')^2}{\tilde{h}}\right) (1-h)}. \quad (4.5)$$

Notice that, just like in [5, 7, 11], we are working in Lorentzian signature and have omitted a factor of i in (4.5).

Regarding y as ‘time’, the fact that the Lagrangian is time-independent implies that the ‘energy’ is conserved, a statement that is easily seen to lead to

$$(r')^2 = \frac{\tilde{h}}{H} \left(\frac{\gamma(1-h)}{H} - 1 \right) = \frac{\tilde{h}}{H} \left(\frac{\gamma r_0^4}{R^4} - 1 \right), \quad (4.6)$$

with $\gamma \geq R^4/r_0^4$ an integration constant. It follows from this equation that the minimum value of $r(y)$, which by symmetry must lie at $y = 0$, occurs at the radius where $\tilde{h} = 0$, i.e., at the horizon r_H .

Integrating (4.6) we find a relation between γ and the width L of the Wilson loop,

$$\frac{L}{2} = \frac{1}{\sqrt{(\gamma r_0^4/R^4 - 1)}} \int_{r_H}^{\infty} dr \sqrt{\frac{H}{\tilde{h}}} = \frac{R^2 I}{r_H \sqrt{(\gamma r_0^4/R^4 - 1)}}, \quad (4.7)$$

where we have defined

$$I = \int_1^{\infty} \frac{d\rho}{\sqrt{\rho^4 + (l^2/r_H^2)\rho^2 - r_0^4/r_H^4}} = \int_0^1 \frac{d\zeta}{\sqrt{1 + (l^2/r_H^2)\zeta^2 - r_0^4/r_H^4\zeta^4}}. \quad (4.8)$$

Also, using (4.6) in (4.5) we are left with a trivial integral that yields

$$S = \frac{L^- L}{2\sqrt{2}\pi\alpha'} \sqrt{\gamma}. \quad (4.9)$$

According to the recipe of [6], to compute the Wilson loop we must subtract from S the self-interaction of the isolated quark and the isolated antiquark, which in the AdS side corresponds to the Nambu-Goto action evaluated for strings that extend from (to) $r \rightarrow \infty$ to (from) $r = r_H$ at fixed y ,

$$S_0 = \frac{\sqrt{2}L^-}{2\pi\alpha'} \int_{r_h}^{\infty} dr \sqrt{\frac{1-h}{\tilde{h}}} = \frac{\sqrt{2}r_0^2 L^- I}{2\pi\alpha' r_H}. \quad (4.10)$$

It is clear from (4.7) that small L corresponds to large γ . In this regime, the leading term in (4.7) implies $\sqrt{\gamma} \propto 1/L$, which when substituted in (4.9) gives an L -independent result that is precisely cancelled by (4.10). The quantity we are after comes from the

next-to-leading term in (4.7) for large γ , which yields

$$S_I = S - S_0 = \frac{r_0^2 r_H L - L^2}{8\sqrt{2}\pi\alpha' R^4 I} . \quad (4.11)$$

Using the definition of [5], the jet-quenching parameter then follows as³

$$\hat{q}_J \equiv \frac{2S_I}{L - L^2/4\sqrt{2}} = \frac{r_0^2 r_H}{\pi\alpha' R^4 I} . \quad (4.12)$$

The final step would be to express the result (4.12) in terms of gauge theory quantities. This can be done analytically in the $l \ll r_0$ regime, where up to $\mathcal{O}(l^4/r_0^4)$ corrections the relations (4.3) and (4.4) imply

$$r_H \simeq r_0 \left(1 - \frac{l^2}{4r_0^2}\right) = r_0 \left(1 - \frac{4J^2}{\pi^2 N^4 T^6}\right), \quad r_0 \simeq \pi R^2 T \left(1 + \frac{4J^2}{\pi^2 N^4 T^6}\right), \quad (4.13)$$

and the integral (4.8) can be seen to give

$$I \simeq \frac{\sqrt{\pi}\Gamma(5/4)}{\Gamma(3/4)} + \frac{l^2}{4r_0^2} (\text{E}(-1) - 2\text{K}(-1)) \equiv a - \frac{l^2}{4r_0^2} b, \quad (4.14)$$

where E and K respectively denote complete elliptic integrals of the second and first kind. The numerical value of the coefficients defined above is $a \simeq 1.311$, $b \simeq 0.7120$. Putting this all together, and remembering that the 't Hooft coupling is given by $\lambda \equiv g_{YM}^2 N = R^4/\alpha'$, we are finally left with

$$\hat{q}_J = \frac{\pi^2 \sqrt{\lambda} T^3}{a} \left[1 + \frac{4J^2}{\pi^2 N^4 T^6} (2 + b/a) + \mathcal{O}\left(\frac{J^4}{T^{12}}\right)\right] . \quad (4.15)$$

As a (rather mild) check, note that the leading term, $\hat{q}_{J=0}$, reproduces the result of [5]. The next-to-leading term is a new result.

Again, it is natural to compare (4.15) against the drag force calculated in [17] (see also [16]),

$$\left(\frac{dp_1}{dt}\right)_J = -\frac{\pi p_1}{2m} \sqrt{\lambda} T^2 \left[1 + \frac{8J^2}{\pi^2 N^4 T^6} + \mathcal{O}(J^4/T^{12})\right] . \quad (4.16)$$

The comparison is especially interesting because the quantities to be compared are now *functions* of the additional parameter J . The leading term in (4.16) is of course the result computed in [14, 15] at zero chemical potential. As in the cascading gauge theory case analyzed in the previous section, we find that the subleading terms in the jet-quenching ratio

$$\frac{\hat{q}_J}{\hat{q}_0} = 1 + \frac{8J^2}{\pi^2 N^4 T^6} (1 + b/2a) + \mathcal{O}\left(\frac{J^4}{T^{12}}\right) \quad (4.17)$$

³We employ here the final normalization of \hat{q} given in v3 of [5]; the original definition was a factor of $\sqrt{2}$ smaller.

and the drag force ratio,

$$\frac{(dp_1/dt)_J}{(dp_1/dt)_0} = 1 + \frac{8J^2}{\pi^2 N^4 T^6} + \mathcal{O}(J^4/T^{12}) \quad (4.18)$$

have the same qualitative form. The numerical coefficients are also in rough agreement: they are both positive, implying that both the drag force and the jet-quenching parameter increase with increasing charge density, and they are of the same order of magnitude.

It is important to note, however, that this agreement cannot persist at arbitrarily high order in the expansion in powers of l/r_0 , because the full result (4.12) for the jet-quenching parameter evidently has a different functional dependence on r_0 and l than the full drag force result [17]

$$\left(\frac{dp_1}{dt}\right)_J = -\frac{r_0^2/R^2}{2\pi l_s^2} \frac{p_1}{m}. \quad (4.19)$$

This appears to imply that, in the general case, the drag force and jet-quenching parameter codify somewhat different information on the process of energy loss in a plasma.

After the first version of this paper had been posted on the arXiv, three other calculations of \hat{q} in a charged $\mathcal{N} = 4$ SYM plasma appeared [35–37]. The first of these is not directly comparable to ours, because the authors of [35] employed a five-dimensional supergravity perspective instead of the ten-dimensional string theory viewpoint adopted here (for a discussion on the relation between these two approaches, see [17]). Our full result (4.12) for the singly-charged plasma agrees with the one obtained by the authors of [36] (who also determined \hat{q} for two equal non-zero charges) and [37] (who addressed the general three-charge case). This result was plotted in [36] and shown to be non-monotonic beyond the small-charge region explored in (4.15), which motivated the authors of [17] to generate a comparable plot of their drag force result (4.19). As expected from the discussion above, the two graphs are different but qualitatively similar, so again the general lesson seems to be that the parameter \hat{q} defined by [5] and the dissipative force extracted from the procedure pioneered in [14, 15] represent closely related but not identical measures of energy dissipation in a given non-Abelian plasma. Evidently, more work will be required to completely elucidate the relation between these two approaches.

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